



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

2. The use of non-statistically trained agents is not necessarily a reflection upon the character of the figures.

3. A large-establishment and large-city index, if properly weighted industrially and geographically, is representative of employment trends for the country as a whole.

4. The geographical influence in the case of employment fluctuation is of slight importance compared with the industrial influence.

5. Although large establishments may be considered representative, care should be taken in the composition of an index to avoid excessively large establishments in cases where they alone control the index as a whole, or significant portions of it.

These statistics are no longer published in the bulletin of the Employment Service, but the machinery for their collection is still in operation in 65 cities and the figures are being returned each month for the use of the Employment Service. For the sake of completion of the record of an important period of employment fluctuation it is to be hoped that they may in time be made available to persons outside the Bureau. With minor changes in the list of establishments returning the figures and with only slight supervision of their collection, it appears probable that these statistics might be made to furnish a thoroughly reliable index of employment fluctuation for the country as a whole, and also reliable indexes both for particular industries and for individual cities. The latter service, which is of increasing importance, is one that is not now afforded by any other series of employment statistics.

THE DETERMINATION OF SECULAR TREND RECONSIDERED

BY HOLBROOK WORKING, *University of Minnesota*

Statisticians will note with pleasure Professor W. L. Crum's discussion of the determination of secular trend in the issue of this JOURNAL for June, 1922. It is one of the welcome signs of the growing interest of mathematicians in the problems with which scientists are struggling in the application of statistical methods in their various fields. The increasing use of statistical methods which economics is experiencing is common to many sciences. In all these sciences progressive workers are finding present knowledge of methods sadly inadequate. The necessity of devoting a large portion of their time to the acquirement of the requisite mathematical knowledge and the development of new and more adequate statistical methods has placed on such workers a heavy burden which is seriously retarding the progress of their work. The

prospect of aid from properly trained mathematicians is cause for rejoicing.

Professor Crum's discussion exhibits a weakness, however, to which the mathematician is naturally prone, and which seriously diminishes the value of his work from the point of view of the statistician. Proposing a method of calculating secular trend which is considerably more laborious than the present method, he neglects to prove that it introduces substantial advantages.

Professor Crum's point, it will be remembered, is that for many purposes the line of secular trend of a time series should be fitted by a method which gives the sum of the squares of the percentage deviations therefrom a minimum. The usual method, of course, gives the sum of the squares of the actual deviations a minimum. The validity of this point is unquestionable, and Professor Crum has made a distinct contribution in providing a method for obtaining the desired fit.

Unfortunately, for the practical statistician, Professor Crum has not gone on to present adequate evidence of the usefulness of his method. Approximations are always acceptable, even in theoretical mathematics, provided they are of a sufficient degree of closeness. It is the opinion of the present writer that the usual method of determining secular trend is such an adequate approximation. In support of this contention the following observations are submitted. It is hoped that they may lead Professor Crum or some other mathematician to provide a more adequate treatment of the subject as it concerns the practical statistician, to whom the labor of extended computations is a large consideration.

The method of *ratios*, to adopt Professor Crum's terminology, is claimed to give a line of secular trend which may differ notably from that obtained by the method of deviations, either in vertical position (*a*) or in slope (*m*) or in both.

The question may be attacked most simply by assuming a line fitted by the usual method, that of deviations, and considering the deviations—the *d*'s—and the ratios—the $\frac{d}{x'}$'s—(*x'* being taken as the ordinate of secular trend corresponding to the deviation *d*). Consider first the values obtained for *a*. By definition we have $\Sigma d = 0$. The value of *a* for the line of secular trend fitted by the method of ratios will differ from that obtained by the method of deviations only in case $\Sigma \frac{d}{x'} = 0$. If the line of secular trend be horizontal, each value of *d* is divided by the same value of *x'* and it follows of necessity that if $\Sigma d = 0$, $\Sigma \frac{d}{x'} \neq 0$. Hence, in this case *a* is the same for both methods.

If the line of secular trend be inclined from the horizontal, it is possible that the positive values of d might in general be associated with somewhat smaller values of x' than the negative values of d . In such a case we would have $\Sigma \frac{d}{x'} > 0$, indicating that a would be larger if obtained by the method of ratios than if obtained by the method of deviations. Similar reasoning shows the possibility of having $\Sigma \frac{d}{x'} < 0$ due to the positive values of d being associated in general with larger values of x' than the negative values of d .

In the absence of rigorous mathematical demonstration the above situations may be accepted as actual possibilities on the basis of the evidence submitted by Professor Crum. However, it is difficult to see how $\Sigma \frac{d}{x'}$ could be greater or less than zero by any considerable amount if the trend is approximately linear. Professor Crum cites three examples in which the values obtained are as follows:

Method	Value of a		
Method of deviations.....	2.9	9.9	398
Method of ratios.....	3.1	10.0	440

There is obviously either a miscalculation or a misprint in the third case as a value of 440 is impossible. The difference in the second case is not clearly significant, since a difference of one unit in the last place may be due to a difference of but one or two units in the succeeding place. The difference in the first case, assuming the computations to have been carried to a sufficient number of places, is significant, subject to two qualifications: (1) it may possibly be due to the fact that the method of ratios introduces an approximation;¹ (2) the series for which the values are computed is too short (seven observations) to be surely typical.

The writer has calculated the value of $\Sigma \frac{d}{x'}$ for seven series for which the data happen to be readily available. The series tested are the values of the annual production of hay, corn, oats, and wheat for 35 years, 1879-1913, as given by Professor Day (the first four of a group of twelve series published in one place,² and three series contained in unpublished material calculated by the present writer. In all but one of the seven cases $\Sigma \frac{d}{x'} = 0$, within the limits of accuracy fixed by the

¹ Cf., *JOUR. OF THE AMER. STAT. ASS'N*, June, 1922, p. 213.

² Cf. *Rev. of Economic Statis.*, September, 1920, p. 258.

number of places carried in the computations. The exception occurs in the case of oats, where $\Sigma \frac{d}{x'} = -.056$, and $\Sigma \frac{d}{x'} = -.0016$. The latter

is the more significant figure, as it is a measure of the vertical displacement of the line of secular trend. The values obtained for $\Sigma \frac{d}{x'}$ for the

seven series are as follows:

Series	Years included	$\Sigma \frac{d}{x'}$ $\bar{N}$
1 (Hay).....	35	-0.0008
2 (Corn).....	35	+0.0002
3 (Oats).....	35	-0.0016
4 (Wheat).....	35	+0.0004
5.....	23	-0.0001
6.....	22	+0.0002
7.....	20	+0.0002

In the case of oats, where the discrepancy is appreciable, the fit given by a straight line is not good; and an error of -0.0016 in its vertical position is insignificant.

The evidence from trial of specific cases, therefore, is distinctly in favor of the assumption that where the secular trend of a series is approximately linear, we shall have $\Sigma \frac{d}{x'} = 0$ (very nearly), and hence that the method of deviations will prove satisfactory in determining the vertical position of the line of secular trend.

Consider now the question of the slope of the line of secular trend. The method of deviations gives Σd^2 a minimum. The method of ratios should give different results only when a different line would give $\Sigma \left(\frac{d}{x'}\right)^2$ a minimum. Under what conditions will Σd^2 be a minimum, but $\Sigma \left(\frac{d}{x'}\right)^2$ be not a minimum?

The answer may be obtained from a consideration of Figure 1. The central group of points are those used by Professor Crum in his second example. The large black circles are at a distance from AB proportioned according to d^2 for the corresponding values of x . The large hollow circles are at a distance from AB proportioned according to $\left(\frac{d}{x'}\right)^2$ for the corresponding values of x . Adopting the concept of the

method of moments, these large circles may be looked upon as exerting a pull upon AB in proportion to their distance therefrom. If the solid circles be considered, these pulls are in equilibrium (Σd^2 is a minimum). But if the large hollow circles be considered, these pulls are not in equilibrium. The forces exerted about the lower end of the line have

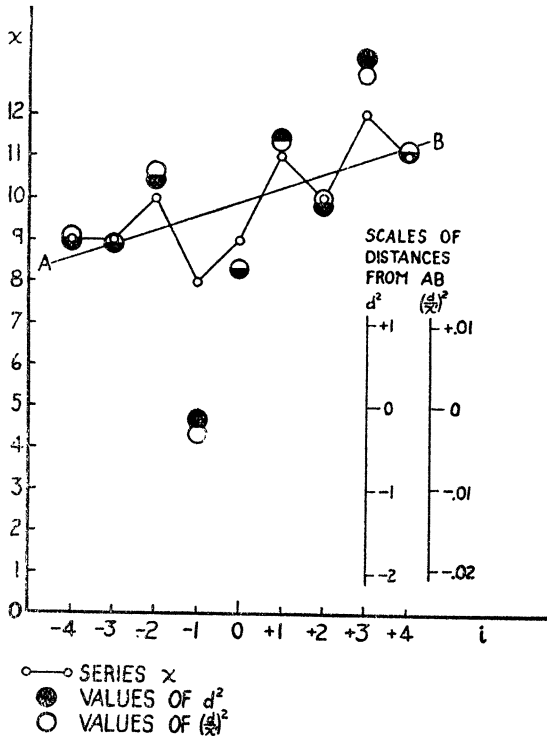


FIGURE 1. AB being placed to give the best fit for actual deviations, it will not in this case give the best fit for percentage deviations.

been increased (as compared with the solid circles), the forces about the upper end of the line have been decreased. It chanced that the forces about the ends of the line are predominantly upward, so there is now a tendency to swing this line about its center. This is the explanation of the different slope obtained by the method of ratios.

Under what conditions will the forces at the ends of the line of secular trend be predominantly in one direction? Only when the trend is not linear. The curved line in Figure 2 is a parabola fitted by the method of least squares to the series shown in Figure 1. Clearly, the trend of the series is far from linear. Only the shortness of the

series prevented this curvature from being immediately obvious. Were the trend of the series truly linear, $\Sigma \left(\frac{d}{x'} \right)^2$ would be a minimum also, and the method of ratios would give the same slope to the line of secular trend as that obtained by the method of deviations.

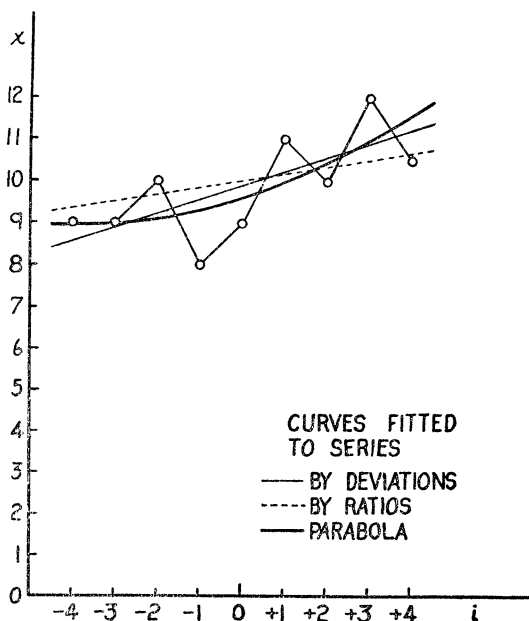


FIGURE 2. The curvature of the parabola shows that the trend of the series is not linear. This is the cause of the difference in slope of the two straight lines fitted to the series.

After a careful consideration of the principles involved, and a test in several specific cases, the following conclusion appears justified:

A straight line fitted by the method of least squares to a time series having an approximately linear trend gives a fit which is almost equally satisfactory whether actual deviations be considered or percentage deviations from the corresponding ordinates of secular trend. Professor Crum's criticism of the usual method is theoretically valid, but of little practical significance.